

# **New and Improved Solar Radiation Models for GPS Satellites Based on Flight Data**

## **Final Report**

Prepared for:

Air Force Materiel Command  
Space and Missile Systems Center/CZSF

Prepared by:

Jet Propulsion Laboratory  
California Institute of Technology

Under Task Order Agreement No. RF-182/808  
Task Plan 80-4193

April 12, 1997

Report Documentation Page				Form Approved OMB No. 0704-0188	
Public reporting burden for the collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to a penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.					
1. REPORT DATE <b>12 APR 1997</b>		2. REPORT TYPE		3. DATES COVERED <b>00-00-1997 to 00-00-1997</b>	
4. TITLE AND SUBTITLE <b>New and Improved Solar Radiation Models for GPS Satellites Based on Flight Data</b>				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S)				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) <b>Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA, 91125</b>				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT <b>Approved for public release; distribution unlimited</b>					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT					
15. SUBJECT TERMS					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT <b>Same as Report (SAR)</b>	18. NUMBER OF PAGES <b>35</b>	19a. NAME OF RESPONSIBLE PERSON
a. REPORT <b>unclassified</b>	b. ABSTRACT <b>unclassified</b>	c. THIS PAGE <b>unclassified</b>			

---

Yoaz E. Bar-Sever  
Task Manager

---

Kenneth M. Russ  
Program Manager

## **New and Improved Solar Radiation Models for GPS Satellites Based on Flight Data**

### **Executive summary**

This report documents the analysis and results from a year-long study of a novel approach to the problem of developing solar radiation models for GPS satellites. The approach is aiming to replace the pre-launch design phase of solar pressure and heat reradiation models by a less costly and more accurate post-launch phase. The approach is also suitable for many other Earth-orbiting satellites. In this approach we exploit the fact that a collection of individual orbit solutions contains more information about the dynamics of the satellite than the information that was used to generate the individual solutions.

The current GPS constellation of Block II and Block IIA satellites was used as a prototype for developing and validating our approach. We have used daily GPS precise ephemerides (produced routinely at JPL for the International GPS Service (IGS)) over a period of 9 months to adjust a parametrized model of the solar pressure so as to obtain best fit. The resulting model proved to be more accurate than the standard solar pressure model for GPS satellites (Fliegel et al., 1992). In a separate effort we developed from first principles a new solar pressure model for eclipsing GPS satellites. This model also represents a significant improvement over the standard model.

The main benefits from the new approach are:

- Cost reduction due to the replacement of costly pre-launch design phase with cheap post-launch process.
- Increased accuracy due to sensing of actual satellite behavior, as opposed to theoretical behavior.
- Increased accuracy by allowing an infinite fine-tuning process.

Increased model accuracy, in turn, will improve:

- Orbit prediction capability including autonav.
- Improved UT1 estimation capabilities
- Improved accuracy for any high-precision GPS application, in particular, scientific applications such as geodesy, meteorology and altimetry.
- Increase our understanding of the space environment and in-orbit satellite behavior.

The most immediate application of the new approach is for the refinement of the nominal solar radiation model for Block IIR satellites (Fliegel and Gallini, 1997) immediately after launch. Model refinement can be made monthly and it is expected that each refinement will be substantial for at least the first year, given the typical BOL anomalies. The greatest utility will be achieved by applying the new approach to the Block IIF satellites. A significant saving in cost can be made by eliminating the pre-launch design phase of the solar radiation model. In addition, lessons learned from the analysis of the Block II solar pressure can be applied to the design of the attitude control system of Block IIF satellites and improve the accuracy of any solar radiation model developed for these satellites.

The work presented here is a proof of concept. Although a significant improvement in the solar radiation model for Block II/IIA satellites was achieved, the case was treated as a prototype and it was impossible in the limited time we had to explore all the possible modeling options. Instead, most of the efforts were spent overcoming the various technical difficulties in implementing our approach and on refining and tuning the technology. Consequently, further

accuracy improvements to Block II/IIA solar radiation models can be realized with only a modest amount of additional analysis.

## Table of Content

<b>INTRODUCTION .....</b>	<b>6</b>
<b>ORBITAL GEOMETRY AND COORDINATE SYSTEMS .....</b>	<b>9</b>
<b>THE APPROACH FOR NON-ECLIPSING SATELLITES.....</b>	<b>10</b>
<b>RESULTS FOR NON-ECLIPSING SATELLITES .....</b>	<b>13</b>
<b>APPROACH FOR ECLIPSING SATELLITES .....</b>	<b>20</b>
<b>RESULTS FOR ECLIPSING SATELLITES .....</b>	<b>21</b>
<b>DISCUSSION .....</b>	<b>23</b>
<b>APPENDIX A. COSINE AVERAGING .....</b>	<b>28</b>
<b>APPENDIX B. ESTIMATED HARMONICS.....</b>	<b>29</b>
<b>APPENDIX C. COEFFICIENTS AND FORMAL ERRORS OF GSPM.II.97 .....</b>	<b>33</b>
<b>REFERENCES.....</b>	<b>34</b>

## Introduction

Orbiting at an altitude of about 20,000 km, with no drag and with limited sensitivity to the details of the Earth's gravitational pull, the GPS satellites seem in little need for a complex dynamical model. Yet the relatively poor geometry of the observation system (providing mostly radial information), combined with the demand of some applications for extremely high accuracy, create the need for a very careful modeling of the forces acting on a GPS satellite.

Of those forces, solar radiation is the most delicate to model (Fliegel et al., 1992). Although the radiation force, at GPS altitude, is only second in magnitude to the gravitational pull from the Sun, Earth and Moon, its uncertainties are much higher (Table 1). The solar radiation model has received relatively little attention since the publication of the T20 model by Fliegel et al. (1992) and is considered to be the largest error source in GPS orbit determination (Bertiger et. al. 1994, Gold et. al. 1995, Beutler et al., 1994). The errors associated with the solar pressure model are particularly large during the satellite's eclipse period (Bar-Sever et al., 1996, Watkins et al., 1996, Fliegel et al., 1992).

Source	Magnitude, m	Uncertainty, %
J2	15,600	0.00001
Moon	880	small
Sun (gravitation)	420	small
Solar pressure	130	1-5
C(2,2), S(2,2)	120	0.001
C(3,m), S(3,m) all m	20	0.01

Table 1. The perturbations on a GPS satellite after one revolution and their uncertainties. The uncertainties in the gravitational forces are based on JGM3 formal errors. The magnitude of the perturbations are based on Fliegel and Gallini (1996).

The application most sensitive to force modeling errors is orbit prediction. This is because, in the absence of constraining measurements, the orbit will typically diverge quadratically from the truth (Colombo, 1989). Certain limits on GPS orbit prediction errors are required by the GPS operators at the US Air Force for proper maintenance of the constellation. Block IIR satellites will have "autonav" capabilities which require quality long-term orbit prediction to allow for the positioning of the satellites in the absence of up-link communications over periods of several months. Many of the recently-emerged real-time applications of the GPS, such as WAAS and weather forecasting, are dependent on the quality of GPS orbit prediction (Bar-Sever, 1996a, Bertiger, 1997). All these applications stand to benefit from improvements to the force models.

As mentioned above, solar radiation mismodeling is a limiting error source in precise applications such as geodetic positioning and low earth orbiter tracking. In particular, systematic errors in modeling the solar radiation alias to GPS-based UT1 rate estimates through the coupling effect of the right ascension of the ascending node. This can cause a uniform rotation of the GPS constellation to be aliased into the UT1 rate estimate (Lichten et al., 1992). Such a uniform rotation can be caused by radiation forces such as the Y bias. Reducing the uncertainties in the radiation force model, especially the Y bias, will contribute to reduce this aliasing and improve the GPS-based estimates of UT1 rate.

The development of a solar radiation model for a spacecraft is usually carried out by the spacecraft manufacturer and is part of their overall contractual obligation. It is to be completed before the spacecraft is launched. This was the case for the Block I, II and IIA satellites for which the ROCK4 and ROCK42 solar pressure models were developed by Rockwell, and it is also the case for the Block IIR satellites developed by Lockheed Martin. The cost of the development of a solar radiation model can reach millions of dollars, depending on the type of the mission. In the case of the Block I, II and IIA satellites, the ROCK models (ROCK4 for Block I and ROCK42 for block II and IIA) were developed by Rockwell, programmed by IBM Federal Systems and revised and improved by Rockwell and The Aerospace Corporation (Fliegel and Gallini, 1996). The best model currently available for Block II/IIA satellites is the T20 model, which is a Fourier series representation of a revised ROCK42 model (Fliegel et al., 1992).

The ground-based design process (as opposed to the flight-data-based design process we are proposing) is carried out by modeling the spacecraft as a collection of components, each with its own shape, size, optical and thermal characteristics. Given the nominal mission profile and orbital geometry, the process employs various ray tracing, finite element and finite difference techniques to simulate the effects of impinging photons on the spacecraft, and derive the solar radiation model. This is an exceedingly complicated process because most spacecrafts have a complicated shape and are made of many different materials. As a results, it is necessary to make some simplifying assumptions in the simulation. For example, some small sub-structures are ignored, optical and thermal properties are approximated (rather crudely), mutual shadowing of components is applied only to the most dominant structures (if at all), and secondary reflections (from one component to the other) are usually ignored. The result is usually a fairly good model - if the spacecraft's in-orbit behavior does not deviate much from the nominal. But the deficiencies of such an approach are clear:

1. The in-orbit satellite behavior may deviate from the nominal. Misorientation, bending and flexing of structures are quite common. For example, the non-nominal Y bias force is attributed to solar array misalignment (Fliegel et al., 1992).
2. It is impossible to gauge the combined effects of all the approximations, simplifying assumptions and outright errors that went into the model. Hence the actual accuracy of the model can only be roughly estimated.
3. The model is not adjustable or tunable. If accuracy requirements change during the lifetime of the mission it usually requires a costly redesign process.

A flight-data-based design process, in contrast, has the following advantages:

1. It is cheap, since all the data is available and is in conventional form, and the models are simple (Fourier expansions).
2. It reflects actual in-orbit behavior.
3. It is more accurate. It directly accounts for the combined radiation pressure of all spacecraft components.
4. It includes radiation effects due to Earth albedo.
5. It is infinitely tunable and adjustable, i.e., once the design process is set up it can be carried out indefinitely and continuously improve and adjust for changing spacecraft and environmental conditions.



6. It provides a tool for learning about actual in-orbit behavior of satellites and for flagging and monitoring problem satellites.

In the following sections we will describe our approach to the design process of flight-data-based solar radiation models for GPS satellites, and we will present evidence to support our claims about the advantages of such a process. As a separate issue we will address the unique problem of the Block II/IIA eclipsing satellites and develop a special solar pressure model for them.

## Orbital geometry and coordinate systems

The GPS satellites occupy six nearly-circular orbital planes with radius of approx. 26,500 km and inclination of  $55^\circ$ . The attitude of the satellites is determined by the requirement that the navigation antennae along the spacecraft-fixed Z axis point toward the geocenter and that the solar array, mounted along the spacecraft-fixed Y axis, be facing the Sun. These two requirements force the spacecraft to yaw constantly, keeping the spacecraft-fixed X and Z axes always in the plane defined by the Earth, spacecraft and Sun. The X axis is required to lie in the half plane containing the Sun and this, except at the two singular points when the Earth, Sun and spacecraft are co-linear, defines the attitude of the spacecraft uniquely. If the spacecraft is assumed to be symmetric around the XZ plane (which is the assumption built into the ROCK models (Porter, 1983)) then the solar pressure is a two-dimensional vector function of a single variable,  $\epsilon$ , the angle between the spacecraft-Sun direction and the spacecraft-fixed Z axis (Figure 1). By definition,  $\epsilon$  is allowed to vary in  $[0^\circ, 180^\circ]$ , but its range during a full orbit revolution depends on the value of the orbit's beta angle. The beta angle is the angle between the spacecraft-Sun direction and the orbital plane (Figure 2). It is defined to be positive if the spacecraft-Sun direction forms an acute angle with the orbit normal and negative otherwise. By definition, beta ranges in  $[-90^\circ, 90^\circ]$  but due to the particular inclination of the GPS orbits it is limited to, approximately,  $[-75^\circ, 75^\circ]$ . The range of  $\epsilon$  at a given beta is  $[|\beta|, 180^\circ - |\beta|]$ . It follows that for high beta angles the solar pressure hardly varies while for low beta angles the solar pressure goes through more of its possible range.

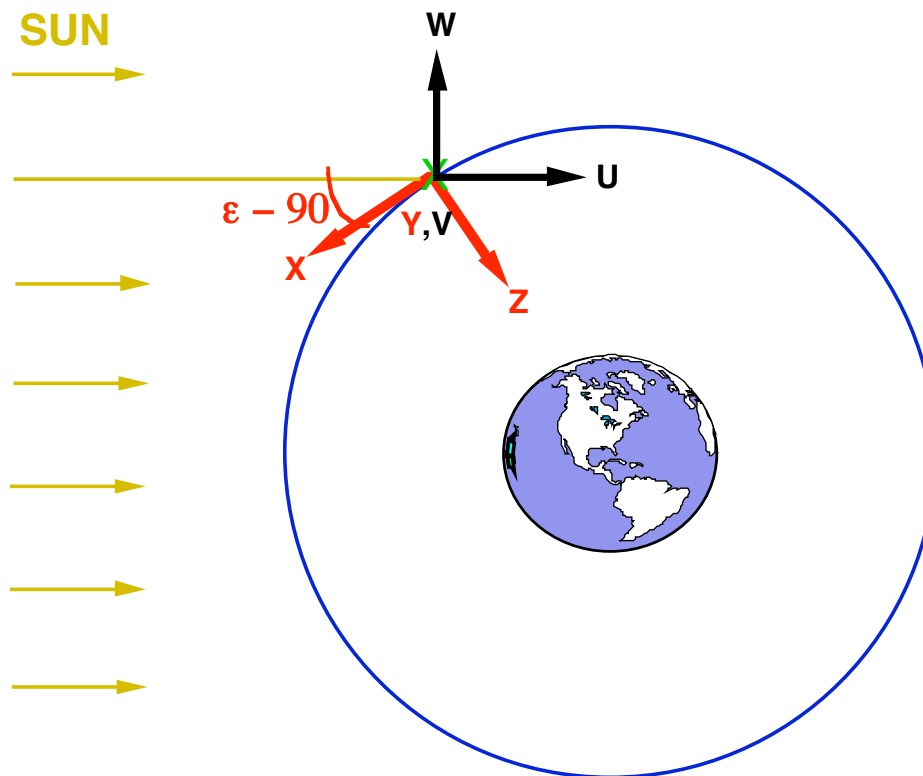


Figure 1. The XYZ and UVW coordinate systems. The orbital geometry corresponds to  $\beta = 0^\circ$  as viewed from above the orbital plane.

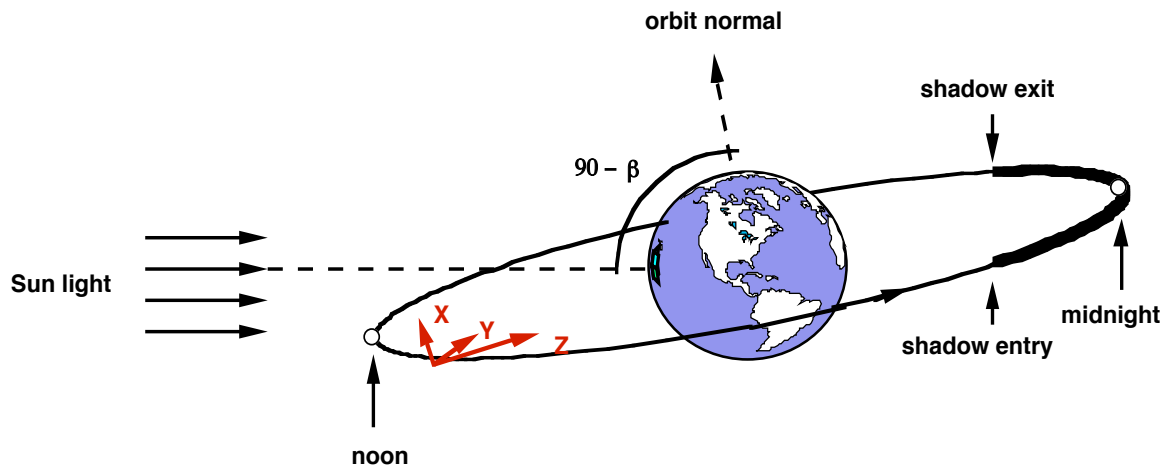


Figure 2. The orbital geometry and the XYZ coordinate system.

The spacecraft-fixed XYZ coordinate system is useful for expressing the solar pressure and, indeed, the ROCK and the T20 models are expressed in this coordinate system. Another useful coordinate system is the UVW system defined as follows: U is pointing along the Sun-spacecraft direction, V is pointing along the cross product between the U direction and the spacecraft-geocenter direction (and is, therefore, identical to the Y coordinate) and W completes the system (Figure 1). Since the bulk effect of the solar pressure is to "push" the spacecraft away from the Sun, it is expected that the dominant component of the pressure will be in the U direction. Consequently, in this system the solar radiation attains a form which is closest to a one-dimensional function and it is, therefore, most natural to express the solar radiation in this system.

### The approach for non-eclipsing satellites

We approach eclipsing and non-eclipsing satellites differently. For the non-eclipsing satellites we use a fully empirical model, the parameters of which are estimated based on a least square fit to "truth" orbits. For eclipsing satellites we used a hybrid approach where a new model, a variant of the T20 model, termed here the "split T20" model, was derived from first principles and a limited set of model parameters was tuned manually. Here are the main elements of our approach for non-eclipsing satellites.

We express the solar radiation model as a truncated Fourier expansion. The expansion variable was chosen to be  $\epsilon$  - the Earth-satellite-Sun angle - since, for GPS satellites following their nominal attitude algorithm, the satellite-Sun geometry is only a function  $\epsilon$ . The expansion is fully parameterized, i.e., the expansion coefficients are presumed unknown a-priori. A satellite trajectory is then created, based on the empirical solar radiation model, and the model parameters are adjusted iteratively to fit the trajectory to a multi-day "truth" orbit in a least square sense. The long-arc "truth" orbit is a key element in our approach. It is the concatenation of multiple daily "truth" orbits and it allows us to retrieve information about the satellite's dynamics that is not present in the individual daily "truth" orbits. The following example illustrates this crucial concept: Imagine having to estimate the mass of a point-like planet around which a satellite is in

an 8-day-period circular orbit. The input is a set of one-day-long “truth” ephemerides that were estimated based on some tracking information and the (erroneous) assumption that the satellite is free of forces (i.e., the satellite has constant velocity). As a result of mismodeling the dynamics, each one-day ephemeris is a straight line, but because they are being constrained by some tracking information they cannot deviate significantly from the actual circular orbit. Concatenating them will create a long-arc “truth” orbit which resembles a circle and allows for the resolution of the parameters of a proper model of the dynamics (see Figure 3). By this analogy, solar radiation mismodeling errors that were built into the daily GPS “truth” solutions can be backed out if we concatenate a sufficient number of these daily orbits and use a properly parameterized model to represent the solar radiation forces.

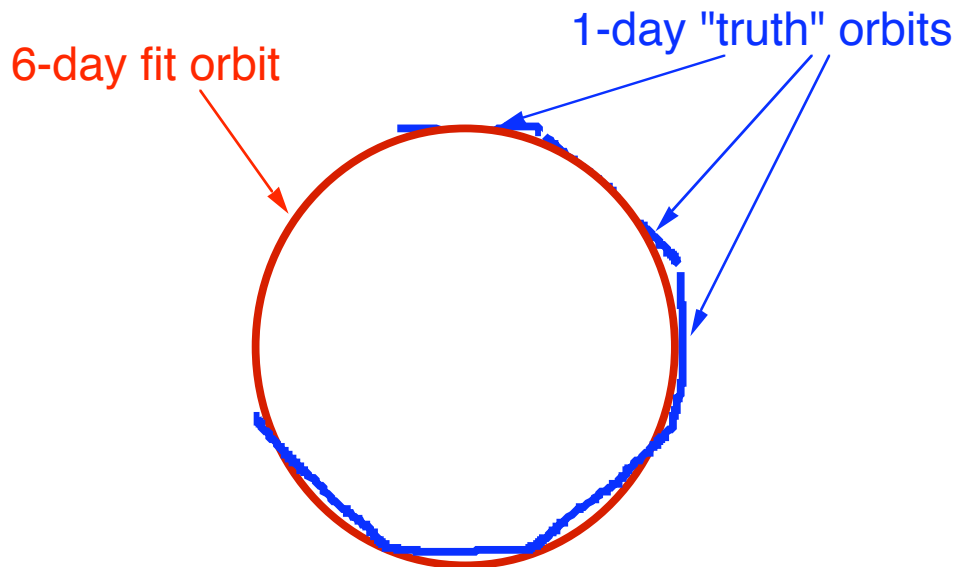


Figure 3. Stringing together straight segments reveals the underlying circle.

In our Block II/IIA prototype system we chose the daily GPS orbit solutions that are produced at JPL for the IGS as our short-arc “truth” orbits. With a 3-dimensional error of about 15 cm RMS, these orbits are of the highest accuracy currently available (Watkins et al., 1997). They are based on the T20 solar pressure model but employ the JPL-developed reduced dynamics technique to mitigate the short-comings of the T20 model and other models by estimating small stochastic accelerations (Zumberge et al., 1995).

Observation of the GPS navigation signal are made in an Earth-fixed system and the most accurate representation of the orbits is available in that system. Since orbit integration is carried out most conveniently in an inertial frame we have to transform the solution from Earth-fixed coordinates to inertial coordinates. To minimize the aliasing of Earth-orientation errors into solar pressure parameters we used the VLBI-based IERS Bulletin-B values of Earth-orientation parameters to transform the solution between the two frames.

The daily JPL solutions span 30 hours, centered around noon of each day. To form a set of daily solutions that can be smoothly concatenated we cosine-average (see Appendix A) each pair of consecutive daily ephemerides and extracted the central 24 hours. We eliminated all satellites that are known to be problematic (SVNs 16, 23, 30, 33, 40) and those satellite-days containing maneuvers, for the cleanest possible data set. The data span is from July, 1995, to May, 1996. The set of transformed, smoothed, truncated and cleaned daily ephemerides form our “truth” orbits. In the non-eclipsing case we also eliminated all eclipsing satellites from the data set.

We can now form very long multi-day arcs by concatenating consecutive daily ephemerides. Our approach calls for estimation of model parameters based on a least-square fit to a multi-day “truth” ephemeris. It is clear that the longer the arc the more information about the dynamics can be retrieved from it and many months of data are needed to resolve the fine details of the solar radiation model. Unfortunately, it is not practical to integrate GPS orbits more than a few days because round-off error will tend to contaminate the integration of the state and variational partials. We found it most practical to perform 10-day integrations. Hence, we had to invent a method to combine estimates based on many 10-day fits. This is not a trivial problem because of the non-linear nature of the estimation problem. This non-linearity also makes it difficult to combine estimates of model parameters for different satellites into one.

### The non-linear estimation problem

Nominally, we assume that all Block II and Block IIA satellites should have identical solar pressure models. This stems from the assumption that they all have the same structure and obey the same attitude laws. But the actual solar pressure is a function of the mass of the satellite and of the solar flux constant. The mass of each satellite is different and both the mass and the solar flux constantly change in time. This makes it difficult to combine estimates across different satellites and different data arcs. Such a combination poses a non-linear estimation problem. For simplicity, assume that the solar radiation force is one-dimensional and is given by:

$$F = \lambda (B_0 + B_1 \cos \varepsilon + A_1 \sin \varepsilon + B_2 \cos 2\varepsilon + A_2 \sin 2\varepsilon + \dots)$$

where  $\lambda$  is a function of the satellite’s mass and of the solar flux constant. Estimating  $\lambda$ , as well as  $B_0$ ,  $B_1$ , etc., from a fit to the 10-day arcs does not always converge, partially because the expansion coefficients are considered completely unknown a-priori. Also, there is no unique solution to this problem. Instead, we chose to estimate  $\lambda B_0$ ,  $\lambda B_1$ , etc., thus postponing the non-linear problem to the combination phase. In this phase we need to combine the estimates  $\lambda_i B_0$ ,  $\lambda_i B_1$ , ... from different satellites and different time periods into a more robust estimate of  $B_0$ ,  $B_1$ , etc.

To describe our solution to this non-linear estimation problem we need a more general notation. Let  $a_i^\alpha$  be a set of estimated parameters where  $i = 1, \dots, n$  labels parameters of the solar pressure model and  $\alpha = 1, \dots, m$  labels distinct 10-day batches and satellites.

We also have the covariances (using summation notation):

$$\langle a_i^\alpha, a_j^\beta \rangle = \delta_{\alpha\beta} C_\alpha^{ij}.$$

We associate a scale factor,  $\lambda_\alpha$ , with each set of estimated parameters, expecting that

$$a_i^\alpha = \lambda_\alpha a_i.$$

We then estimate  $\lambda_\alpha$  and  $a_i$  to minimize

$$J = \frac{1}{2} \sum (a_i^\alpha - \lambda_\alpha a_i) (C_\alpha^{-1})^{ij} (a_j^\alpha - \lambda_\alpha a_j),$$

where the explicit sum is over  $\alpha$ . Again, this is a non-linear problem but notice that  $\lambda_\alpha$  can be solved analytically as a function of  $a_i$  from the  $m$  equations:  $\partial J / \partial \lambda_\alpha = 0$ . When this is substituted back into  $\partial J / \partial a_i = 0$  we obtain a system of  $n$  (highly) non-linear equations for the  $n$  unknowns  $a_i$ . We solve this system numerically. The redundancy of the system is eliminated by fixing

$$a_1 = E[a_i^\alpha].$$

With this choice our numerical scheme always converged. It is not important that the estimated values of  $a_i$  obtained in this scheme contain an arbitrary scale since any precise realization of a GPS solar radiation model must include the estimation of a scale factor. The only thing that is required of the estimates,  $a_i$ , is to be “reasonable” and the choice of  $a_1 = E[a_i^\alpha]$  seems to guarantee that. As a sanity check we verified that the estimates for  $\lambda_\alpha$  are all within a few percent

of 1, in agreement with our knowledge of time-variations in the solar flux constant and variations in mass properties between satellites. There is no practical limit on the number of satellites or 10-day batches that can be combined with this scheme.

Evaluation of solution quality is the last step in our approach. This is done by testing each model's ability to best-fit a pre-selected set of truth ephemerides. There are four types of tests:

1. best fit to single-day "truth" arcs,
2. best fit to 4-day "truth" arcs,
3. best fit to 10-day "truth" arcs,
4. fit to "truth" during a 4-day prediction period based on best-fit to previous 4-day "truth" orbit.

Each type of test consists of many (~100) satellite-arcs to provide a robust evaluation of the model's performance. Although the model's parameters are fixed to their estimated value during the test run there is a set of auxiliary parameters that are always being estimated to provide best fit, in addition to the six epoch state parameters. Not least among these is the overall scale factor which, as mentioned above, always needs to be estimated to account for the small but undetermined scale that is present in the model's estimated parameters. Other auxiliary parameters vary depending on the type of the model being tested but they usually include the Y bias and, perhaps, one or two additional parameters that are deemed not constant in time or between satellites. The significance of the various test types increases in the order they are listed above. Since we usually adjust at least 8 auxiliary parameters (six epoch state, overall scale and Y bias) our ability to fit short orbit arcs is largely affected by errors in our "truth" ephemerides rather than the quality of the estimated model. The test that is most sensitive to the model's quality is the prediction fit. In this fit we fix the value of the auxiliary parameters to those estimated for best fit in the 4-day test case. We then continue to integrate the orbit for four more days and compare the last 4 days of the trajectory to the "truth". The intention here is not to generate the best prediction possible (this will require a different estimation scheme during the first four days) but, rather, to compare the performance of various models in a relative sense.

In summary, our approach consists of 4 steps:

1. obtaining daily "truth" orbits spanning many days and forming 10-day arcs by concatenation,
2. for each 10-day arc and satellite adjusting the parameters of a Fourier-like solar radiation model to best fit the "truth" orbit,
3. combining the estimates from each 10-day arc and for each satellite, together with their full covariance matrix into a single, robust estimate for the model's parameters,
4. evaluating the resulting model (with the estimated parameters) by comparing against "truth" ephemerides.

## Results for non-eclipsing satellites

The model space for the possible solar radiation model is infinite. For the purpose of this prototype development we have limited our search to very low order Fourier expansions. We chose the UVW coordinate system for the expansion because we expect the fewest Fourier terms in this system. We postulate the expansion:

$$F_u = f(m,r) s (C\_U_0 + C\_U_1 \cos \varepsilon + S\_U_1 \sin \varepsilon + C\_U_2 \cos 2\varepsilon + S\_U_2 \sin 2\varepsilon + \dots)$$

$$F_v = C\_V_0 + C\_V_1 \cos \varepsilon + S\_V_1 \sin \varepsilon + \dots$$

$$F_w = f(m,r) s (C\_W_1 \cos \varepsilon + S\_W_1 \sin \varepsilon + C\_W_2 \cos 2\varepsilon + S\_W_2 \sin 2\varepsilon + \dots)$$

where  $f(m,r)$  is a function of the satellite's mass and distance from the Sun.  $s$  is an overall scale factor that is set to 1 during the estimation of the model parameters. Notice that the V structure is different than the U and W structures. That is because the nominal symmetry of the spacecraft about the UW plane leads to consideration of forces orthogonal to this plane as anomalous and

fewer terms are expected in the  $V$  structure. Indeed, until this study revealed energy in higher harmonics,  $C_{V_0}$  was considered the only significant component (since the  $V$  coordinate is identical to the  $Y$  coordinate,  $C_{V_0}$  is just another name for the  $Y$  bias). We experimented with a few low order truncations of the above expansion. For a given truncation, all of the harmonics are estimated using 10-day batches spanning 9 months and all “good”, non-eclipsing satellites. Since the overall scale,  $s$ , was set to 1, these estimates effectively contain an unknown scale. The scale may vary between different satellites and different 10-day arcs but is the same for all estimated harmonics in a given 10-day arc and a given satellite.

Figures 4-6 illustrate the range of the estimated values, together with their formal error for a few of the most interesting harmonics. These figures demonstrate the subtleties of the estimation process. For low beta angles, the  $U$  coordinate lies close to the orbital plane and the  $F_u$  component of the solar pressure is dynamically significant (Colombo, 1992). As the beta angle increases, the dynamic significance of the  $F_u$  harmonics diminishes. This is reflected by the high formal errors for  $C_{U_0}$  as beta increases in absolute value (Figure 4). Figure 5 depicts the estimated values of  $C_{V_0}$ . For high beta angles the  $V$  coordinate points along-track and is, therefore, dynamically very significant. For low beta angles, the  $V$  coordinate rotates about the radial direction (the  $Z$  coordinate), and because it does not point strictly across-track it is also dynamically significant. This fact is reflected as low formal errors for all  $V$  harmonics at all beta values. One of the most interesting results of this study is the discovery of significant energy in the first cosine harmonic in the  $V$  direction. Figure 6 depicts the estimated values of this harmonic. It shows a remarkable coherence in the estimated value between all satellites and a very low level of formal error. Plots of other parameters are presented in appendix B.

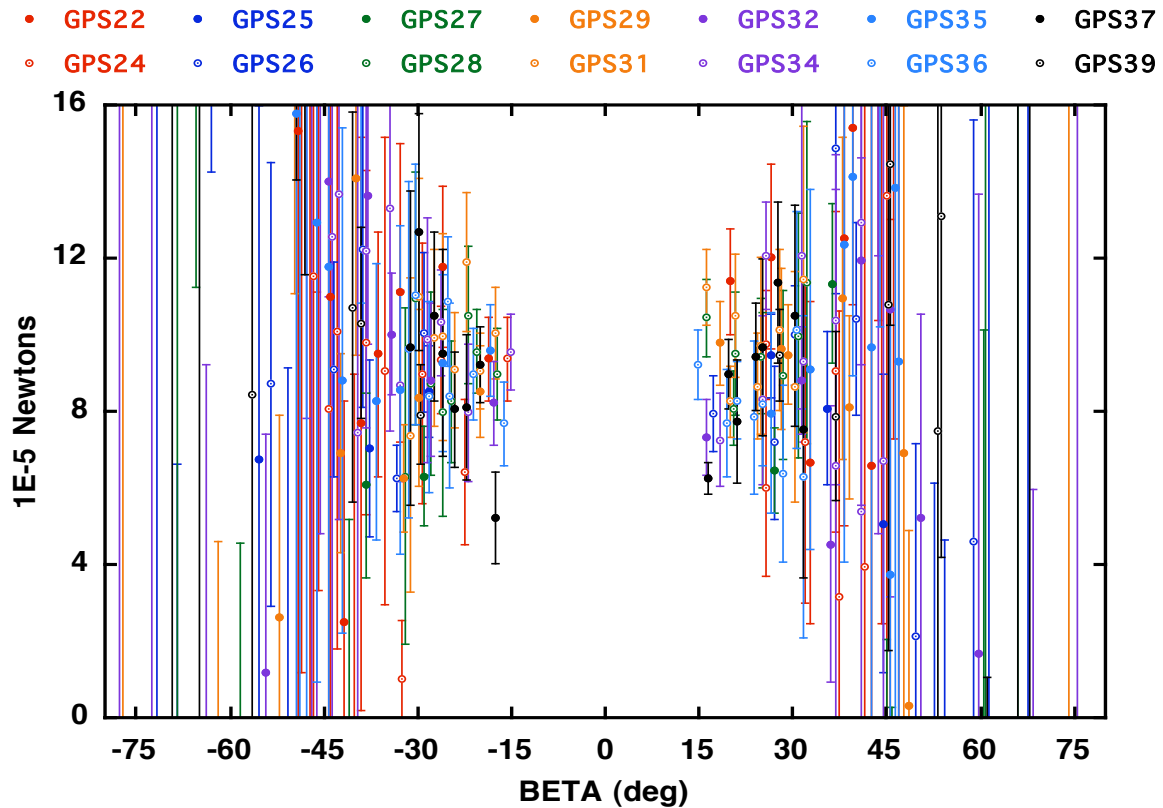


Figure 4. Estimates of  $C_{U_0}$ , with formal errors for non-eclipsing ( $|\beta| > 14$ ) Block IIA satellites. The estimates for high beta angles are usually off the scale of this plot.

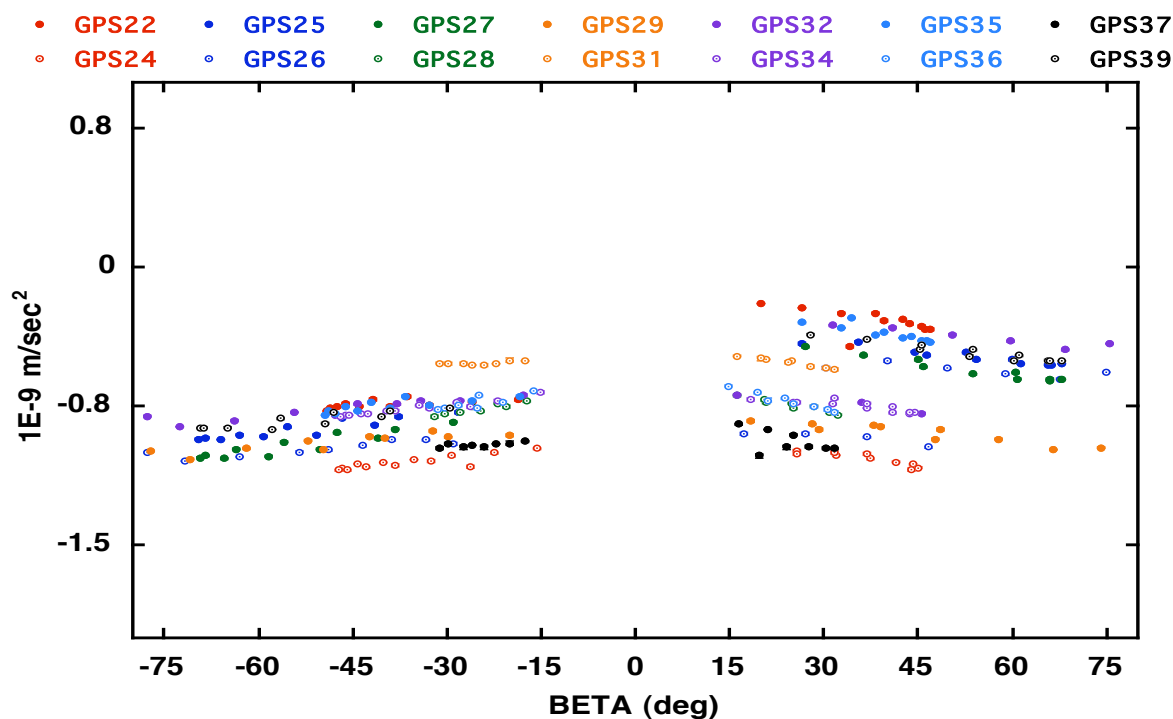


Figure 5. Estimates of  $C_{V_0}$  (Y bias), with formal errors for non-eclipsing ( $|\beta| > 14$ ) Block IIA satellites. The formal errors are usually too small to be seen.

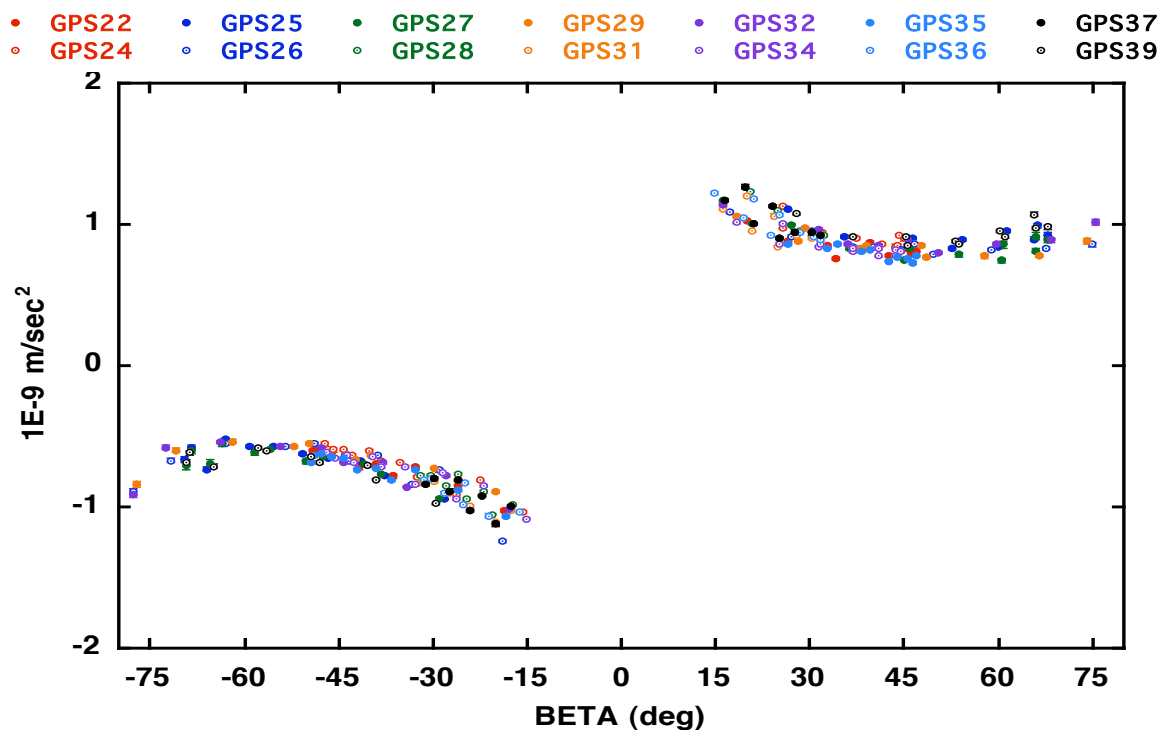




Figure 6. Estimates of  $C_{V_1}$ , with formal errors for non-eclipsing ( $|\beta| > 14$ ) Block IIA satellites . The formal errors are usually too small to be seen.

After we obtain the set of estimated values for each harmonic from all satellites and 10-day arcs, we can combine them to form a single, robust estimate for each harmonic. At this point we need to select the set of harmonics that, first, are assumed constant after removing the satellite- and arc-specific scale and, second, are dynamically important. Here some trial and error is required to pick a good set of harmonics to determine our model. We have chosen not to merge the  $C_{V_0}$  estimates because they are traditionally always being estimated and, being anomalous, we suspect that they might not be scaled the same way as the U and W harmonics. The estimates of  $C_{V_1}$  clearly demonstrated that this harmonic is not a constant and it was also omitted from the combination process. After some experimentation the following set of harmonics was chosen for the combination process:

$C_{U_0}$ ,  $C_{U_1}$ ,  $S_{U_1}$ ,  $S_{U_2}$ ,  $S_{W_1}$ ,  $C_{W_1}$  and  $S_{W_2}$ . Although not constant, our analysis showed that the presence of the  $C_{V_1}$  harmonic improves significantly the quality of the test fits. Consequently, we approximated the estimated values of  $C_{V_1}$  as the following function of beta (see Figure 7):

$$C_{V_1} = 0.1 + 0.5 \sin \beta + 0.3/\sin \beta$$

The impact that the presence of  $C_{V_1}$  makes on the quality of the model is illustrated in Figure 8.

Our final model takes the form (full precision and formal errors are reported in appendix C):

$$F_u = f(m,r) s (8.98 - 0.15 \cos \varepsilon + 0.71 \sin \varepsilon + 0.1 \sin 2\varepsilon)$$

$$F_v = v\_bias + (0.1 + 0.5 \sin \beta + 0.3/\sin \beta) \cos \varepsilon$$

$$F_w = f(m,r) s (-0.03 \sin \varepsilon + 0.73 \cos \varepsilon - 0.74 \sin 2\varepsilon)$$

where  $s$  and  $v\_bias$  are always estimated following the conventional practice. This model is labeled GSPM.II.97+CY<sub>1</sub>. If the  $C_{V_1}$  harmonic is excluded the resulting model is labeled GSPM.II.97. This model is as simple in structure as the T20 model and it has only a few more coefficients. The T20 model is:

$$F_x = f(m,r) s (-8.96 \sin \varepsilon + 0.16 \sin 3\varepsilon + 0.1 \sin 5\varepsilon - 0.07 \sin 7\varepsilon)$$

$$F_y = y\_bias$$

$$F_z = f(m,r) s (-8.43 \cos \varepsilon)$$

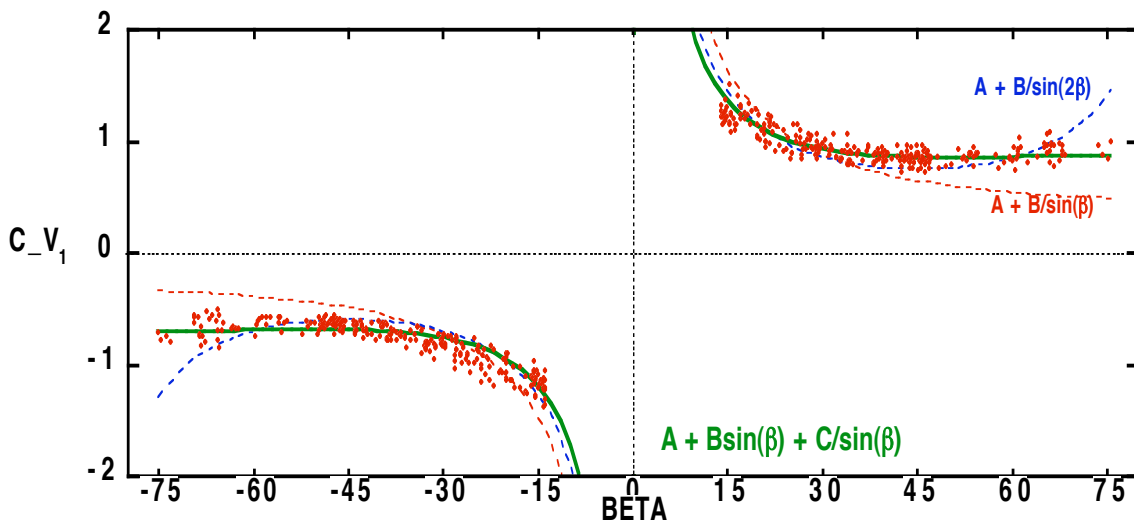


Figure 7. Fitting a functional form to  $C_{V_1}$ .

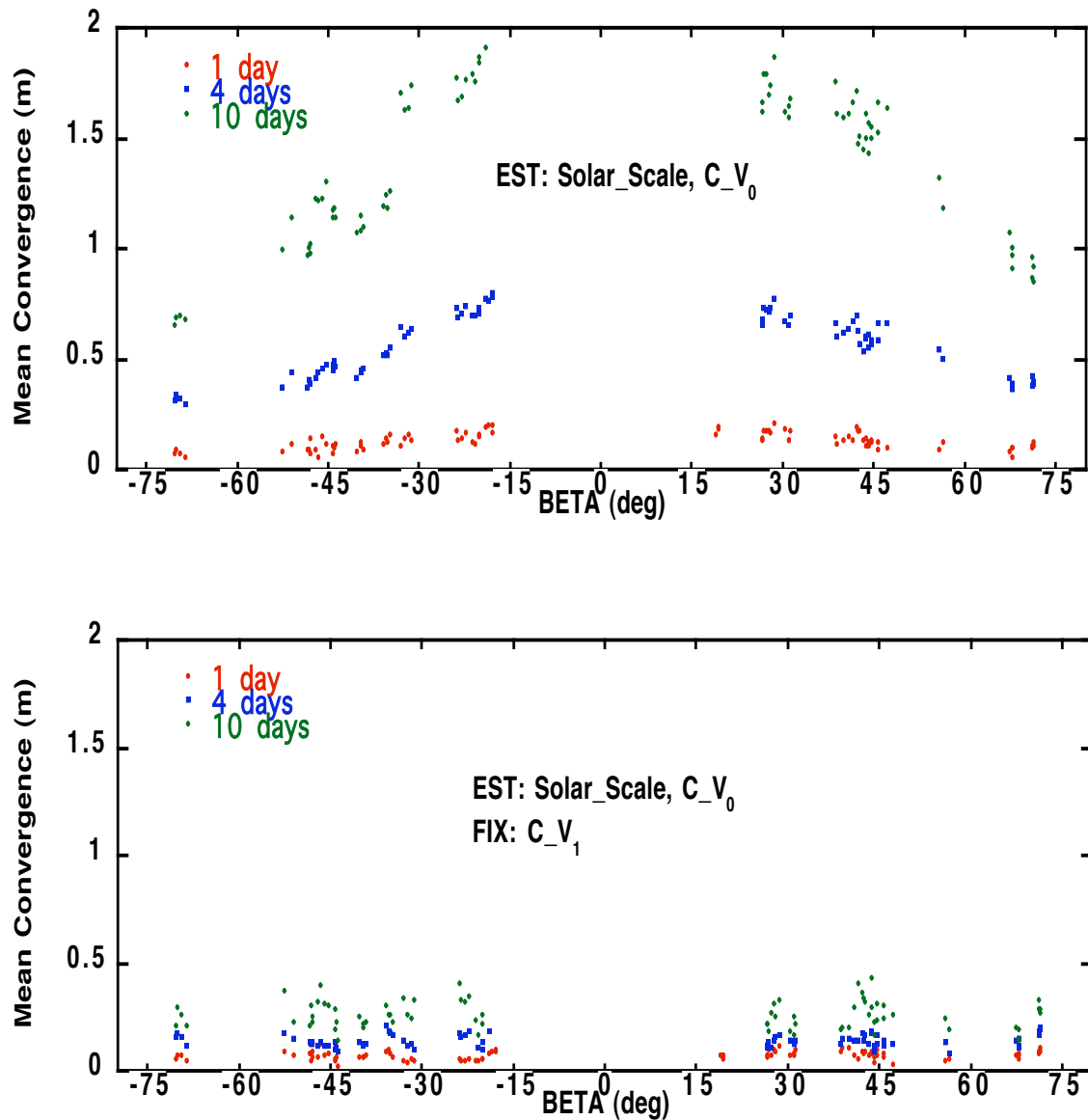


Figure 8. Effects of the  $C_{V_1}$  harmonic on the ability of the model to fit the 1- 4- and 10-day “truth” ephemerides. Top: the GSPM.II.97 model with additional estimation of solar scale and  $C_{V_0}$  (Y bias). Bottom: same as in top panel but with the addition of  $C_{V_1}$  to the model.

The performance of our new model is summarized in Figure 9. Most of the improvement in model performance is due to the presence of the  $C_{V_1}$  harmonic. But even if this harmonic is omitted, our flight-data-based model performs as well as or better than the T20 model.

We have experimented with various other low order Fourier expansions expressed both in the UVW and XYZ coordinate systems. The differences in overall accuracy were found to be negligible and, consequently, they are not presented here.



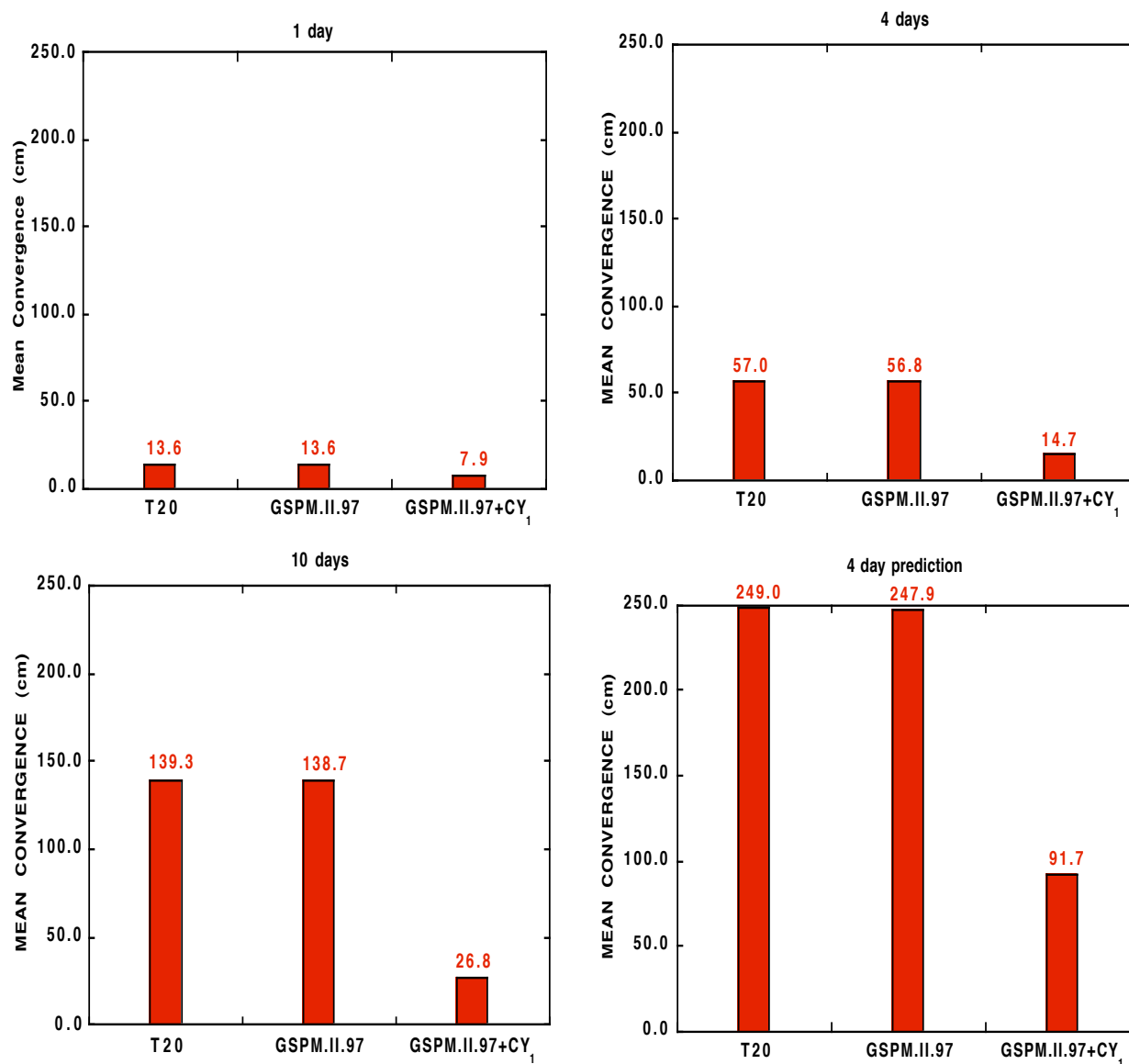


Figure 9. Performance of the flight-data-based model in comparison to the T20 model. The performance is measured in terms of mean RMS fit to “truth” ephemerides. In all cases solar pressure and Y bias ( $C_{V_0}$ ) are the only estimated parameters.

## Approach for eclipsing satellites

The problem in modeling solar pressure for eclipsing GPS satellites stems from the fact that GPS Block II/IIA satellites display a non-nominal attitude behavior during eclipse season. The non-nominal behavior is limited to the yaw axis and to regions of the orbit around “noon” and “midnight”. Orbit noon is defined as the point on the orbit that is closest to the Sun. Orbit midnight is defined as the point on the orbit that is farthest from the sun. The nominal attitude calls for the satellite’s Z axis (the yaw axis) to point toward the geocenter and for the satellite X axis to lie in the Sun-Earth-satellite plane and point in the general direction of the Sun. Also, the large solar array is assumed to be directly facing the sun. The T20 model erroneously assumes that GPS satellites maintain nominal attitude at all times. The non-nominal behavior manifests as a non-nominal pointing of the X axis which, in turn, leads to mispointing of the solar array. This happens during every shadow crossing and up to 45 minutes thereafter, and in the vicinity of orbit noon at low beta angles ( $< 5^\circ$ ) for up to 30 minutes. This non-nominal pointing is the largest error source in the solar pressure model for eclipsing satellites. (For background on the eclipsing problem please refer to Bar-Sever et al., 1996 and Bar-Sever, 1996). The actual attitude of an eclipsing satellite can be modeled during all phases of the orbit and is described in Bar-Sever (1996). The actual attitude model has been part of the GIPSY-OASIS software since September, 1994, but the lack of a compatible solar pressure model prevented its use in the dynamic modeling of the satellite’s trajectory until now.

Our approach for developing a solar pressure model for Block II/IIA eclipsing satellites differs from our approach for non-eclipsing satellite for the following reasons:

1. the orbital geometry of eclipsing satellites is non-periodic,
2. future GPS blocks (Block IIR and Block IIF satellites) will behave differently during eclipse periods,
3. the errors in the current model (T20) are much larger for eclipsing satellites than they are for non-eclipsing satellites.

Item number 1 suggests that the Fourier expansion approach would be much more difficult to implement for eclipsing satellites, requiring possibly another expansion variable. Item number 2 suggests that methods used in developing solar pressure for Block II/IIA eclipsing satellites cannot be readily used for Block IIR and IIF eclipsing satellites. Item number 3 suggests that even a crude model might improve upon the currently-available model. Consequently, we approached the problem of improving the solar pressure model for eclipsing satellites not as a prototype for future implementation but as a means for itself. To this end we proceeded to develop a force model from first principles.

We assume that when the satellite is nominally oriented we can use the nominal solar pressure model. This can be the T20 model or one of our newly developed models. For simplicity we based this development on the T20 model. We further assume that during shadow crossing no radiation forces are acting on the satellite. What is left, is to design a model which will be valid during the non-nominal pointing periods, which occur mainly after shadow exit. During this post-shadow maneuver the satellite is yawing rapidly to regain its nominal yaw attitude which was lost during shadow crossing (Bar-Sever, 1996).

We termed the new model “split T20” for reasons that will be obvious soon. To derive it we assumed that the total force indicated by the T20 model is a superposition of two contributions: a radiation force contributed by the solar array and a radiation force contributed by the main body of the spacecraft. To determine the two contributions we proceeded as follows. Let  $F_x$  and  $F_z$  be the X and Z components of the T20 force in the X and Z coordinates. In the UVW coordinate system the

T20 force has components  $F_u$  and  $F_w$  were,

$$F_u = -F_x \sin \epsilon - F_z \cos \epsilon = 8.43 + f(\epsilon)$$

$$F_w = F_x \cos \epsilon - F_z \sin \epsilon = g(\epsilon)$$

We focus on the constant contribution to  $F_u$ . Solar array radiation forces are nominally constant because the solar array is nominally always facing the Sun. Hence the solar array radiation force constitutes part of the 8.43 constant. But the body radiation force should also have a zero-order term that will contribute toward the 8.43 constant. We now introduce a new parameter,  $r$ , equal to the fraction the solar array contribution is from 8.43. Based on available information about the satellite structure, the value of  $r$  should be approximately 0.9. Given  $r$ , we have *split* the T20 model into its two components: a constant solar array radiation,

$$F_u = r \cdot 8.43,$$

and the remainder, due to main body radiation, given by

$$F_u = (1 - r) \cdot 8.43 + f(\epsilon)$$

$$F_w = g(\epsilon).$$

Once we split the T20 model into its two components we make the following additional assumptions: 1. The satellite's body is XY-symmetric and, 2. The radiation force on the solar panels obeys the rule:

$$F = -C \cos \alpha [(2/3) \delta + 2 \rho \cos \alpha] N + (1 - \rho) S]$$

where  $C$  is a function of the area to mass ratio,  $\alpha$  is angle between the spacecraft-Sun direction,  $S$  and  $N$  - the normal to the solar array.  $\delta$  and  $\rho$  are the diffusive and specular reflectivity, respectively, of the solar array. Given these assumption we simply rotate the main-body-induced radiation force from its nominal direction around the  $Z$  axis by the amount of yaw error. The solar array contribution is now split along the  $S$  and  $N$  directions according to the above formula.  $C$ , which is not known a-priori is determined by setting  $N = S$  and  $F = r \cdot 8.43 \cdot S$ . Notice that during nominal orientation  $N = S$  and the "split T20" model reduces to the T20 model.

The biggest challenge here is the proper modeling of  $N$ , the solar panel normal, as a function of time. Following the principles of the post-shadow maneuver outlined in Bar-Sever (1996), we added a model to our GIPSY software that accounts for the motion of the solar array, assuming a pitch rate of 0.25 deg/sec. A critical element in this model is the determination of the sense of yaw during the post-shadow maneuver. As discussed in Bar-Sever (1996), this can be very difficult whenever the yaw error after shadow crossing approaches  $180^\circ$ . This issue has not been resolved yet and, consequently, the performance of the model is sub-optimal. We estimate that about 10% of the post-shadow maneuvers are modeled incorrectly as a result of this deficiency. This is still a significant improvement over the 100% of post-shadow maneuvers mis modeled by the T20 model.

## Results for eclipsing satellites

The model has three parameters:  $r$ ,  $\delta$  and  $\rho$ . Their values were determined manually using trial and error so as to achieve best fit to the "truth" ephemerides. The chosen values are:

$$r = 0.9.$$

$$\rho = 0.25.$$

$$\delta = 0.3.$$

The performance of the "split T20" model was compared to that of the T20 model in the same way as was done for the non-eclipsing satellites, namely, by comparing their ability to fit "truth" ephemerides of various lengths. The results as summarized in Figure 10, demonstrate clearly the very significant improvement realized by the "split T20" model. In this comparison  $C_{Y1}$  ( $C_{V1}$ )

is being estimated in all cases in addition to the solar scale and Y bias. The singular behavior exhibited by  $C_{Y_1}$  at low beta angles (Figure 7) precludes us from using its functional form as a fixed part of the model, as we did for the non-eclipsing satellites.

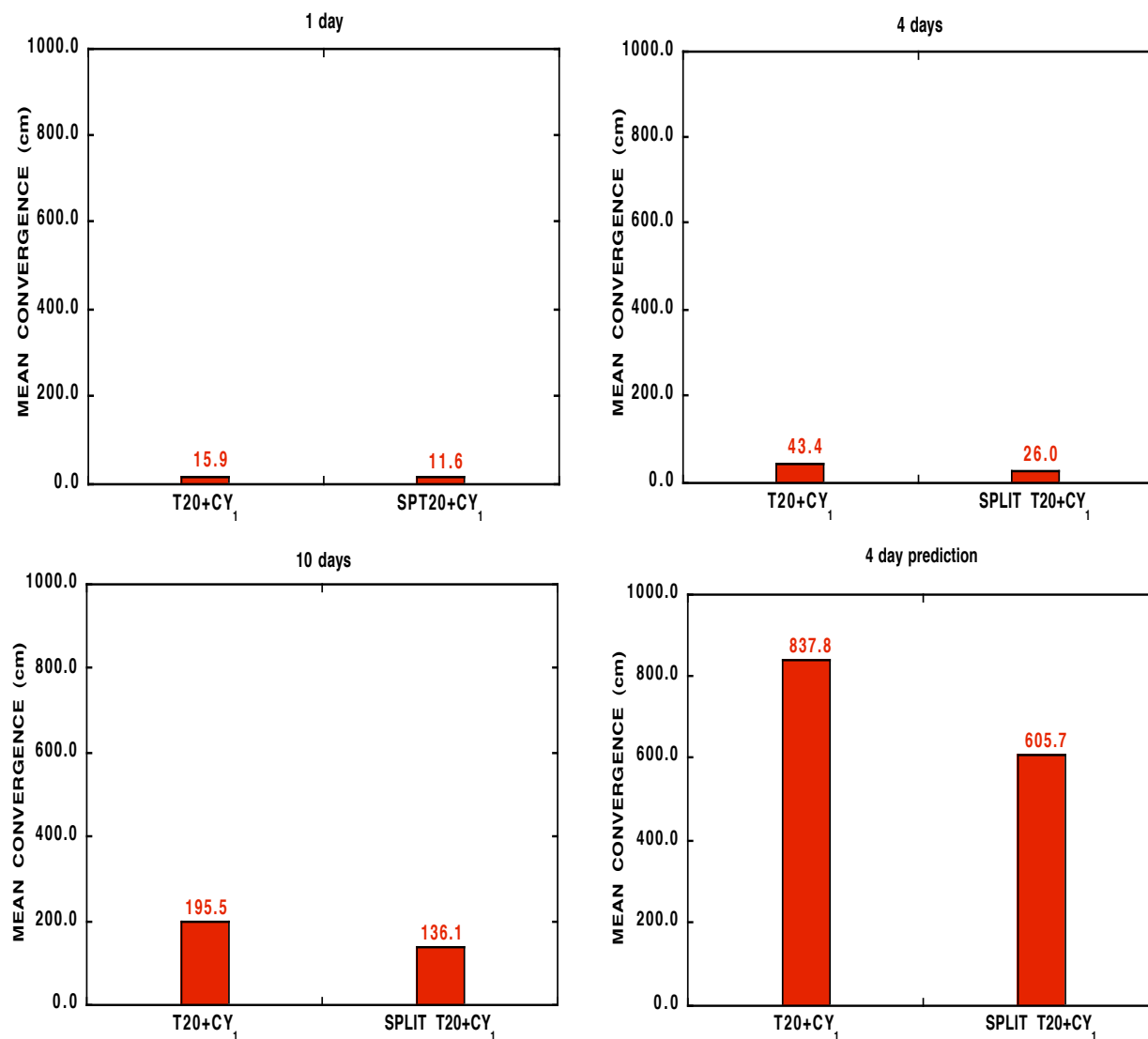


Figure 11. Performance of the “split T20” model in comparison to the T20 model. The performance is measured in terms of mean RMS fit to “truth” ephemerides. In all cases, overall scale, Y bias ( $C_{Y_0}$ ) and  $C_{Y_1}$  are the only estimated parameters.

## Discussion

We have demonstrated here the successful development of a flight-data-based solar pressure model for non-eclipsing GPS Block II/IIA satellites. Although the scope of this work did not allow for an exhaustive search for the best possible model, a significant improvement over the T20 model was achieved with the limited set of models we tested. The models are all in the form of a Fourier expansion in  $\epsilon$ . Almost all of the improvement in model quality comes from a newly-found structure in the Y (V) direction which we were able to express as a simple function of  $\epsilon$  and  $\beta$ . This structure can be easily embedded in any existing solar pressure model to yield significant improvement. The term “solar radiation” should not be construed as accounting only for radiation emanating directly from the Sun. In fact, the flight-data-based model cannot distinguish between Earth radiation in the optical (albedo) and infra-red range and direct solar radiation. Instead, it lumps them together, which is part of its strength. The term “solar radiation model” is still appropriate since the ultimate source for all radiation effects is the Sun.

In terms of overall accuracy we have not found any significant difference between the two coordinate systems we experimented with, although, as expected, the UVW system was more “economical”, requiring fewer terms in the expansion. It is noteworthy, though, that given our limited model space we found it difficult to improve significantly upon the T20 model’s XZ components but, also noteworthy, is the ease with which we could get a similar level of accuracy.

It is evident from the level of error still present in orbit prediction (Figure 9) that there is room for further modeling improvements. Since our model space in this study had to be limited to low order Fourier expansions we might conclude that further improvements can be realized by incorporating higher order terms. The significant magnitude of the newly discovered first order term in the Y direction seems to suggest the possibility of higher order structure in that direction. Feeding more data into our low order models had only a marginal effect. Improvements may also be realized from tuning the model for individual satellites although more data may be needed to compensate for the increase in degrees of freedom.

The ability of the new models to *improve* GPS orbits (rather than *fit* them) from raw observations still needs to be demonstrated. It is expected that iterating on this process will result in further improvement to the model and to the model-based determined orbits.

An important by-product of this development effort has been the insight into the dynamics of GPS satellites afforded by examining highly accurate, quality-controlled, long time series of GPS ephemerides in a well defined inertial frame. To our knowledge this is the first time that such high-quality data has been produced and examined. Our preliminary analysis revealed, for the first time, hard evidence relating the Y bias to solar array misalignment - a long-standing hypothesis (Fliegel et al., 1992). Figure 5 displays a baffling asymmetry in the behavior of the Y bias estimates as a function of beta. Rearranged as a function of time, a large discontinuity is revealed in the estimates for seven satellites, SVNs 22, 25, 26, 27, 32, 35 and 39 (Figure 11). The common denominator for all these satellites is that the yaw bias implemented in their attitude control system changes sign simultaneously with the observed discontinuity. Prior to November, 1995, the GPS operators had routinely changed the sign of the yaw bias in the satellite attitude control system when the orbit’s beta angle switched sign. Bar-Sever (1996) suggested a link between this practice and a simultaneous jump that was observed in the values of the estimated yaw rates. From November 1995 onward, the yaw bias was set permanently to  $+0.5^\circ$  and indeed, since then, no discontinuity was observed either in the estimated yaw rate or in the values of the estimated Y bias. This connection suggests that the yaw bias is partially responsible for the currently observed Y bias. It is by no means the only contributor to the Y bias but it does suggest



that even a small amount of solar array misalignment, on the order of  $1^\circ$ , is a likely cause of the Y bias.

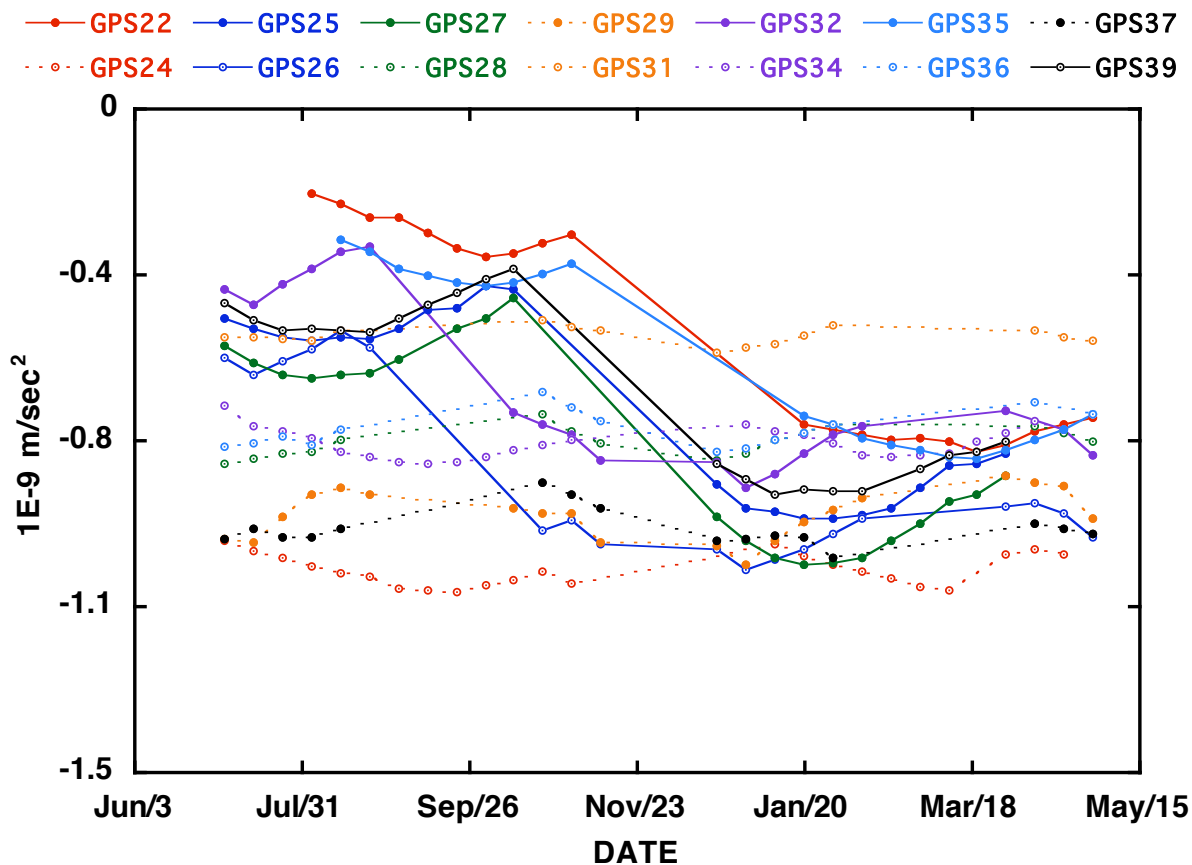


Figure 11. Estimated values of Y bias as a function of time for Block IIA satellites. The estimates are based on 10-day fits to “truth”. The satellites displaying discontinuities across eclipse season are denoted by the heavy lines. The satellites not displaying the discontinuity are denoted by the broken line. The time span is 1995 - 1996.

Significant improvement to the modeling of eclipsing satellites was realized through the “split T20” model. This model still uses the T20 model during the portion of the orbit when the satellite is oriented nominally (about 85% of the time). Much of the remaining error is due to unresolved post-shadow maneuver ambiguities. Although it is possible, in principle, to resolve all the post-shadow maneuver ambiguities given enough flight data, it is impossible to do so in the case of orbit prediction, when no such data is available. This is the main reason why the model for eclipsing satellites will never be as accurate as for non-eclipsing satellites.

One of the perennial problems with GPS has been the resolution of UT1 rate. In precise GPS orbit determination solar radiation parameters as well as length-of-day are being simultaneously adjusted. Unfortunately, a significant level of correlation between the Y bias and UT1 rate contributes to errors in the realization of an inertial frame by the GPS. One of the unique advantages of our technology is the ability to estimate Y bias in a “solid” inertial frame defined by VLBI. Figure 5 displays these estimates, per satellite, as a function of the beta angle. These estimates are based on 10-day fits over 9 months. Ignoring the discontinuities explained earlier, the estimates exhibit remarkable coherence per satellite. These estimates can be the basis of a Y bias function that is made part of the solar pressure model. This will eliminate, or significantly

reduce, the need to estimate Y bias in normal precise orbit determination operations and increase the quality of the UT1 rate estimates.

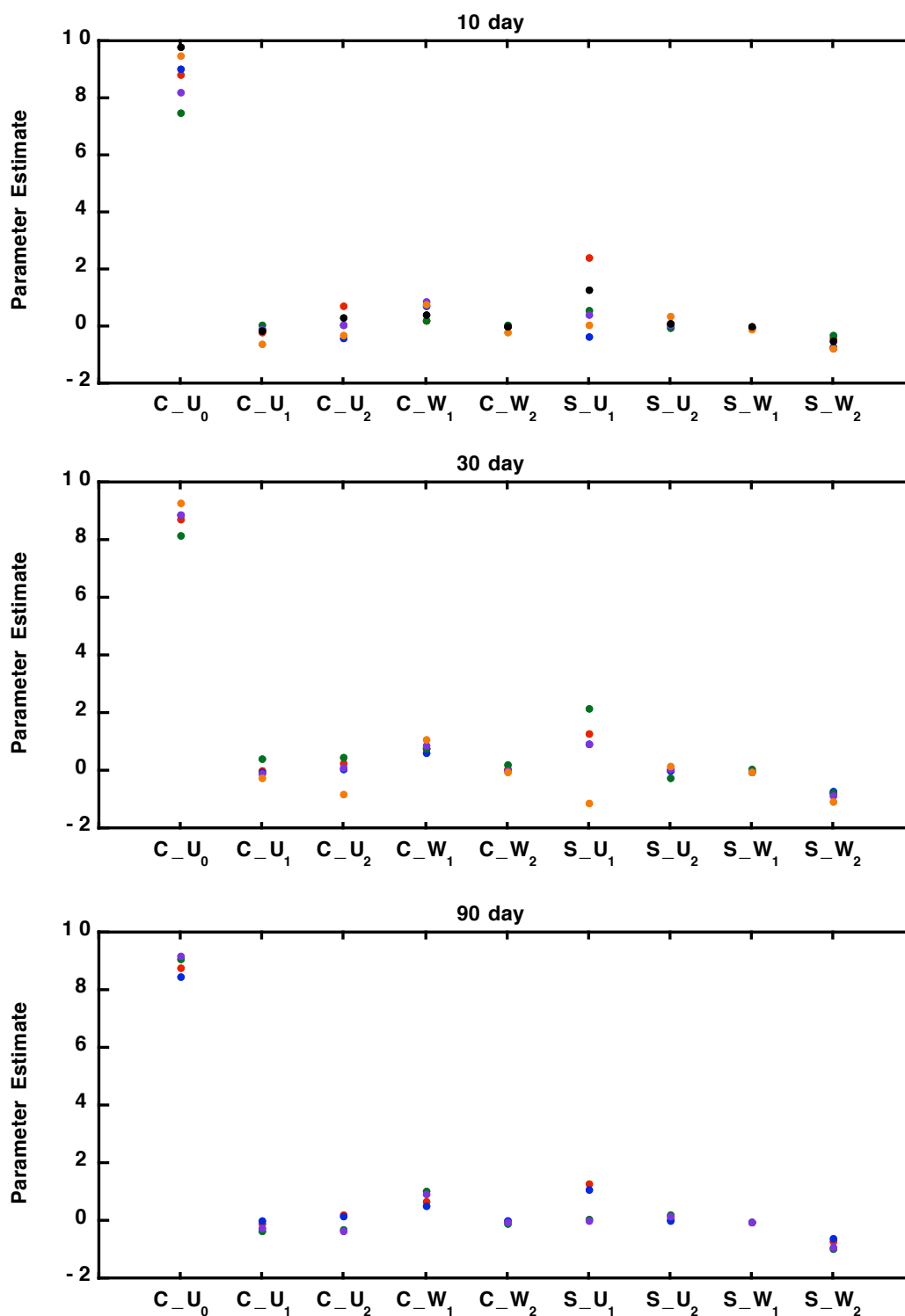


Figure 12. Estimates of model parameters based on data arcs of various lengths. Each point represents an estimated value of the corresponding parameter from a single arc. The top panel is for estimates based on 10-day arcs, the middle panel is for estimates based on 30-day arcs and the bottom panel is for estimates based on 90-day arcs.

In order to demonstrate the usefulness of this technology for newly-launched satellites, where the amount of data is very limited initially, we studied the relationship between the quantity of the data and the quality of the estimates. To this end, we set up an experiment where we estimated the model parameters based on 10 days of data, 30 days of data, and 90 days of data. The results presented above rely, of course, on 9 months of data. In this experiment we divided the 9 months to disjoint sets of 10-, 30- and 90-day arcs and we estimated the model parameters from a fit to each arc. The effect on the estimated values is illustrated in Figure 12. As can be expected, the scatter in most model parameters decrease when more data is brought to bear. Nevertheless, if a model which is based on a single 10-day fit is used during the 10 day period immediately after that, its ability to fit a truth orbit is similar to that of a model based on 9 months of data. The main advantage of having more data is to make the model more robust, namely, valid throughout all mission phases. This result suggests that this technology is suitable for use with newly launched satellites, where the robustness of the model can be increased gradually as more data become available, and as preliminary models are refined in an iterative process.

An ideal opportunity to validate and to tune this technology for newly-launched satellite will come as Block IIR satellites are about to be launched. Block IIR satellites follow a different attitude control algorithm than Block II/IIA satellites. This requires some preparatory work to model the new algorithm. We expect that application of this technology immediately after launch will give rise to a solar radiation model with much better accuracy than the T30 model (Fliegel and Gallini, 1996) and, in particular, will be able to account properly for the expected BOL anomalies.

The ultimate benefit from this technology will be derived from applying it to Block IIF satellites. Since the pre-launch design phase of the solar radiation model has not begun yet, it is possible to realize significant cost savings by eliminating it altogether. Another possible benefit for Block IIF satellite can be reaped if the attitude algorithm design process, currently taking place, will avoid the limitation on accuracy for eclipsing satellites imposed by Block II, IIA and IIR designs. As mentioned above, the dynamics of Block II/IIA (and we expect also Block IIR) eclipsing satellites can never be modeled with the same high fidelity as non-eclipsing satellites. This is not unavoidable, and solutions do exist that eliminate this problem.

## Appendix A. Cosine averaging

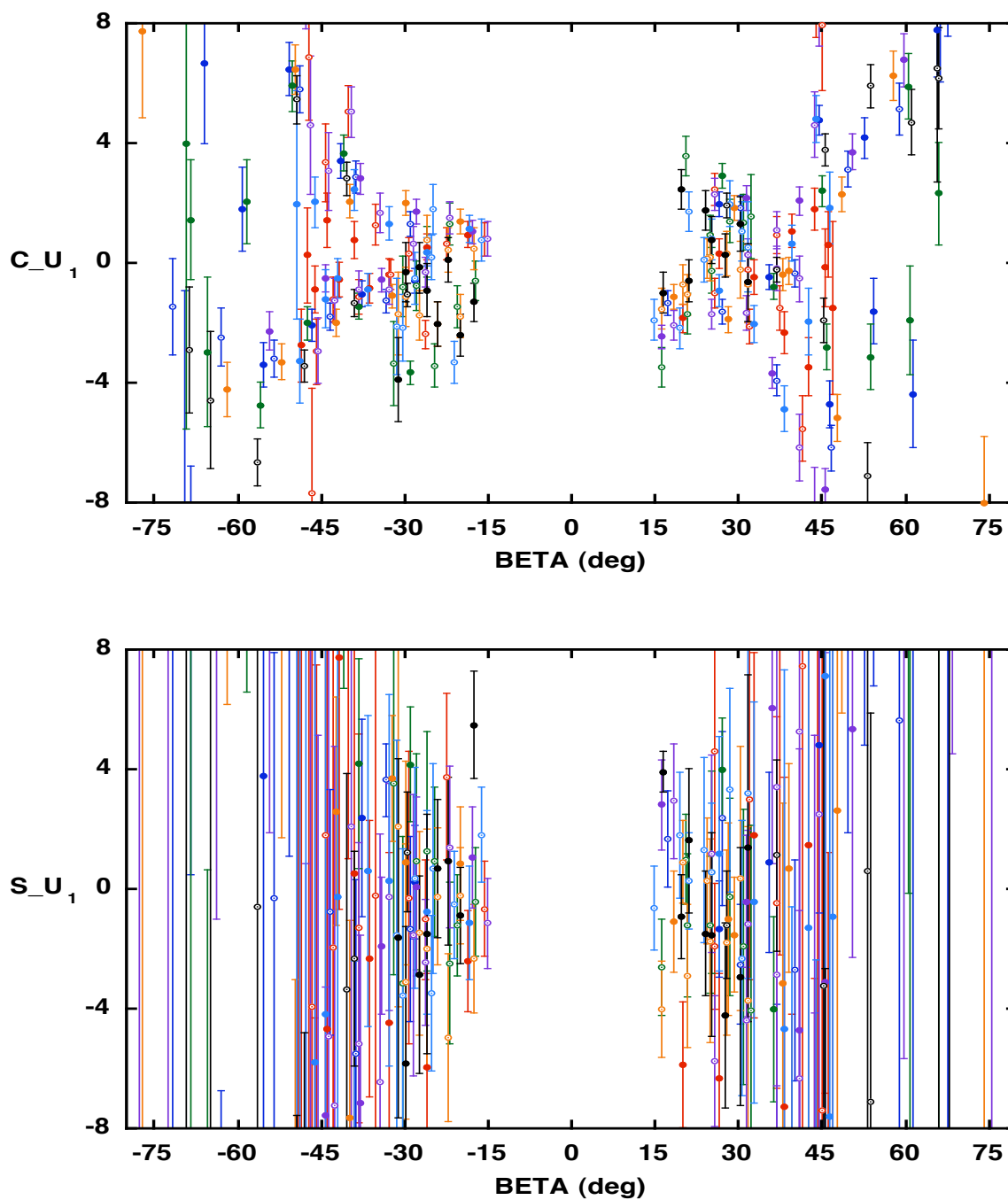
Given two 30-hour ephemerides with 6-hour overlap, this scheme averages the data points in the overlap region by giving them variable weight based on their distance from the edge of the relevant 30-hour span. If A is a data point from the last 6 hours of the first day and B is the corresponding data point from the first 6 hours of the second day, then the cosine averaging of A and B is

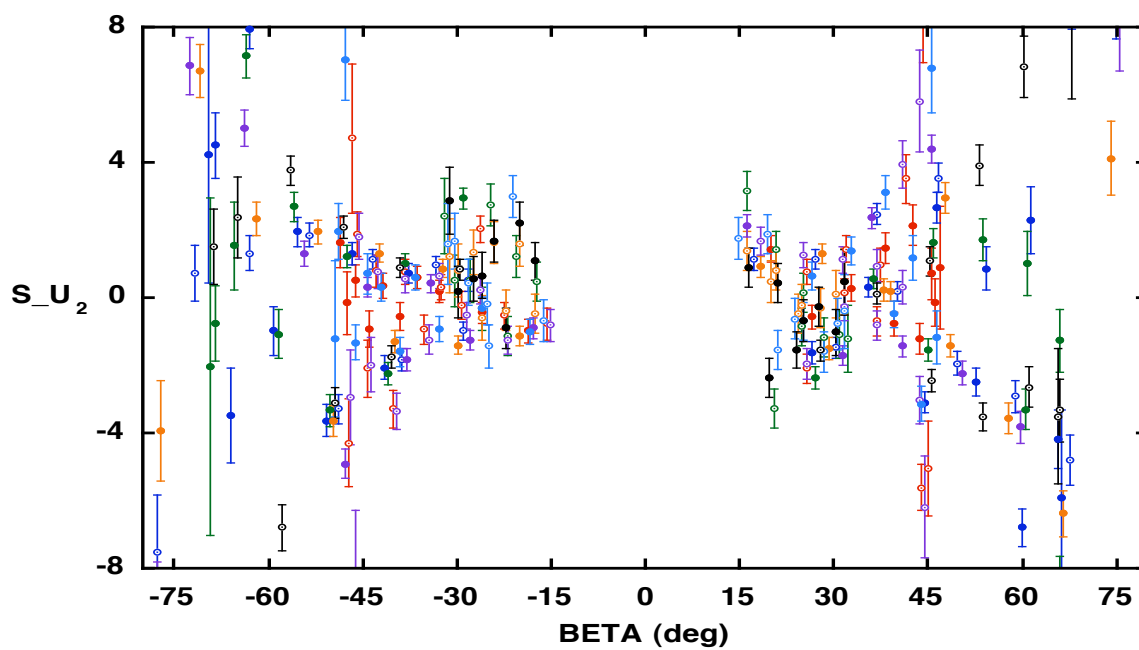
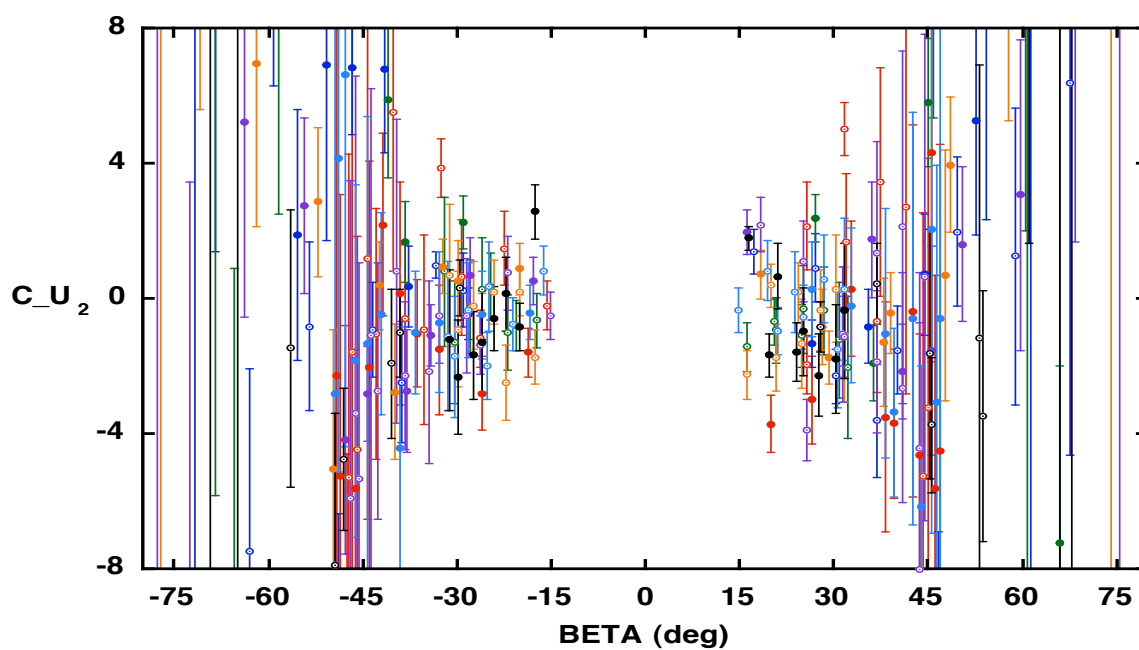
$$[A + B] = A (1 + \cos(\pi T/6))/2 + B (1 - \cos(\pi T/6))/2,$$

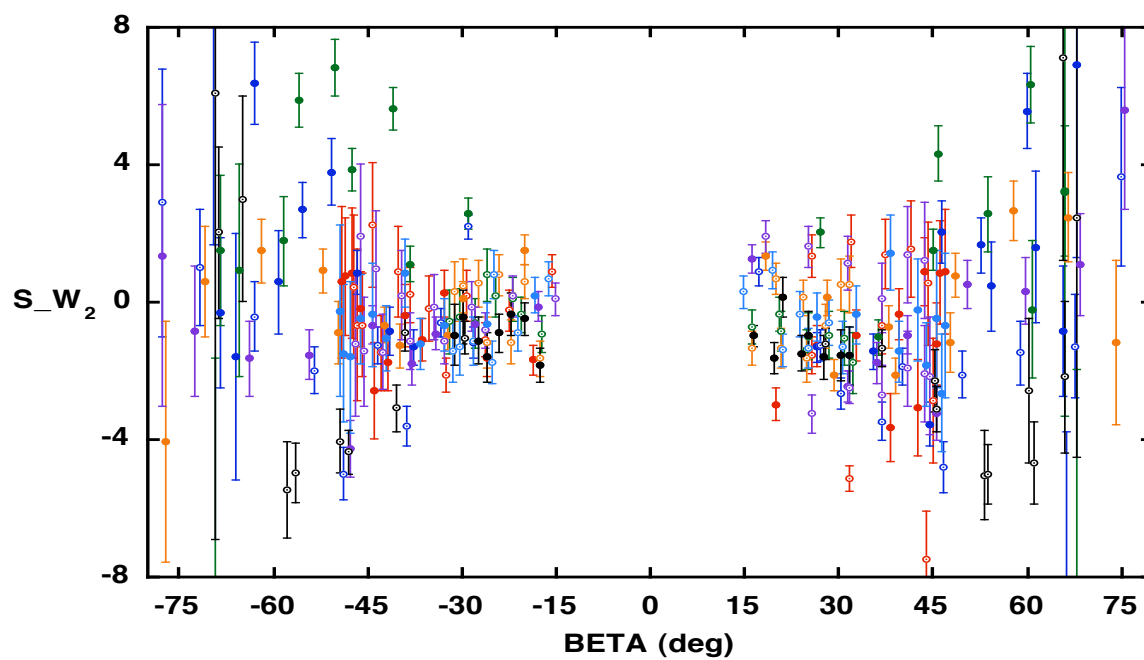
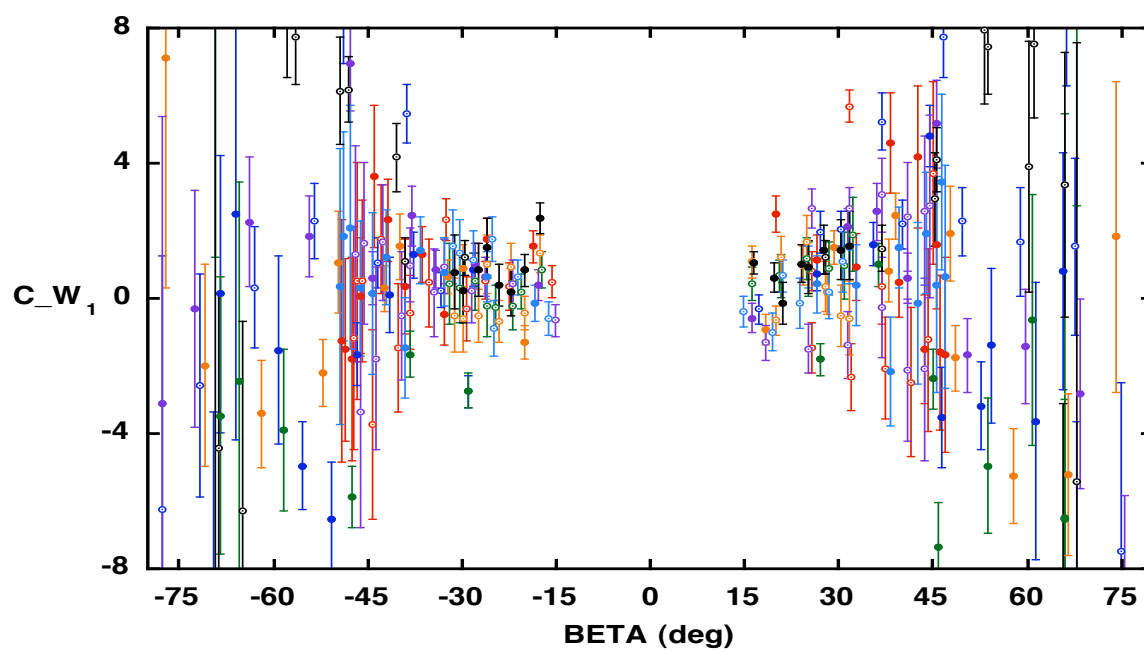
where T is the time distance, in hours, between data point A and the beginning of the overlap period.

## Appendix B. Estimated harmonics.

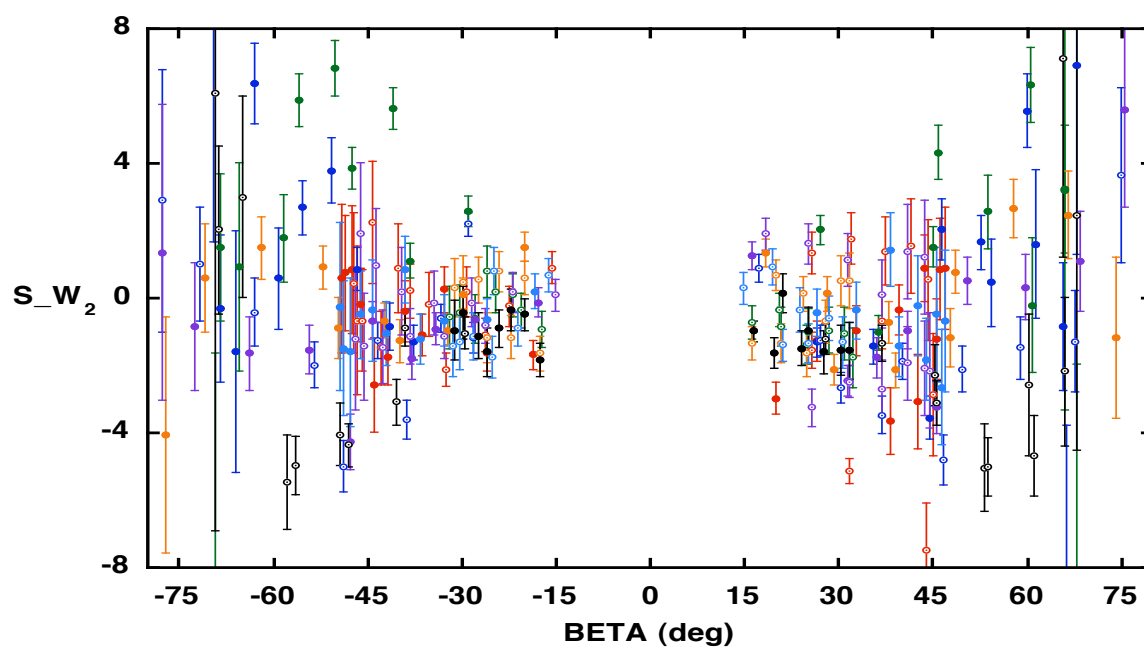
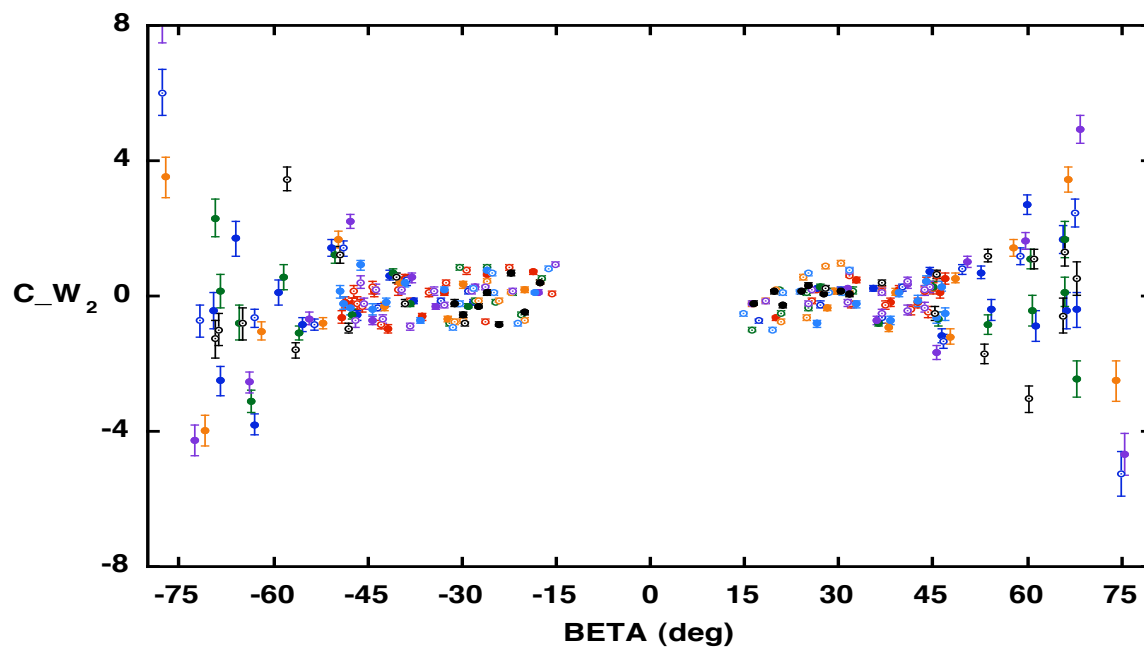
The Figures in this Appendix depict the estimated values and formal errors for all the parameters of the GSPM.II.97 model not given above. The abscissa scale is always  $10^{-5}$  Newtons.











**Appendix C. Coefficients and formal errors of GSPM.II.97**

$C_{U_0} =$	8.981090 $\pm$	0.020854
$C_{U_1} =$	-0.148668 $\pm$	0.009257
$C_{U_2} =$	-0.000274 $\pm$	0.013199
$S_{U_1} =$	0.713423 $\pm$	0.032058
$S_{U_2} =$	0.101430 $\pm$	0.005688
$S_{W_1} =$	-0.025771 $\pm$	0.002933
$S_{W_2} =$	-0.742966 $\pm$	0.010121
$C_{W_1} =$	0.732856 $\pm$	0.014482
$C_{W_2} =$	-0.005984 $\pm$	0.002590

## References

- Bar-Sever, Y.E., A New Model for GPS Yaw Attitude, *Journal of Geodesy*, 70, pp 714-723, 1996.
- Bar-Sever, Y.E., Strategies for Near Real Time Estimates of Precipitable Water Vapor, *Proceedings of the IGS workshop*, Silver Springs, MD, May, 1996a.
- Bar-Sever, Y.E., J. Anselmi, W.I. Bertiger, E.S. Davis, Fixing the GPS Bad Attitude: Modeling GPS Satellite Yaw During Eclipse Seasons, *Navigation*, Vol. 43, No. 1, spring, 1996.
- Bertiger, W.I., Y.E. Bar-Sever, B.J. Haines, B.A. Iijima, S.M. Lichten, U.J. Lindqwister, A.J. Mannucci, R.J. Muellerschoen, T.N. Munson, A.W. Moore, L.J. Romans, B.D. Wilson, S.C. Wu, T.P. Yunck, G. Piesinger, M. Whitehead, A Prototype Real-Time Wide Area Differential GPS System, *Proceedings of the ION National Technical Meeting*, Santa Monica, CA, January, 1997.
- Bertiger, W.I., Bar-Sever, Y.E., Christiansen, E.J., Davis, E.S., Haines, B.J., Ibanez-Meier, R.E., Jee, J.R., Lichten, S.M., Melbourne, W.G., Muellerschoen, R.J., Munson, T.N, Vigue, Y., Wu, S.C., Yunck, T.P., Schutz, B.E., Abusali, P.A.M., Rim, H.J., Watkins, M.M., Willis, P., GPS Precise Tracking Of Topex/Poseidon: Results and Implications, *J. Geophys. Res. Topex/Poseidon Special Issue*, Vol. 99, No. C12, 1994.
- Beutler, G., Brockmann, E., Gurtner, W., Hugentobler, U., Mervart, L., Rothacher, M., Verduin, A., extended Orbit Modeling Techniques at the CODE Processing center of the International GPS Service for Geodynamics (IGS): Theory and Initial Results, *manuscripta Geodaetica*, 19, pp. 367-386, 1994.
- Colombo, O.L., The Dynamics of Global Positioning Orbits and Determination of Precise Ephemerides, *Journal of Geophysical Research*, Vol. 94, No. B7, pp. 9167-9182, 1989.
- Fliegel, H.F., Gallini, T.E. and Swift, E., Global Positioning System Radiation Force Models for Geodetic Applications, *J. Geophys. Res.*, 97, No. B1, 1992.
- Fliegel, H.F. and Gallini, T.E., Solar Force Modeling of Block IIR GPS Satellites, *J. of Space Craft and Rockets*, in press, 1997.
- Fliegel, H.F., Gallini, T.E., Swift, E.R., Global Positioning System Radiation Force Model for Geodetic Applications, *J. Geophys. Res.* Vol. 97, No. B1, pp 559-568, January 10, 1992.
- Gold, K et. al., 1995. Precision Orbit Determination in the GEOSAT Orbit. *Proceedings of The 1995 ION Technical Meeting*, Anaheim, California.
- Heflin, M. et. al., 1994. GPS Time Series: January 22,1991 - April 1, 1994. IOM 335.4-94-008 (JPL internal document), Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA.
- Lichten, S.M., et al., 1992. Sub-Daily Resolution of the Earth Rotation Variations With Global Positioning System Measurements. *Geophys. Res. Let.*, Vol. 19, No. 6.

Porter, W.W., Solar Force-Torque Model for the GPS Space Vehicle System, Rockwell International, TOR76 MA641, enclosure 2 (Feb. 1976), Opt. Model Rept. CDRL 064A14, Seal Beach, CA, 1976, reissued, 1983.

Watkins, M.M, Bar-Sever, Y.E., Yuan, D-N, Evaluation of GPS orbital Ephemerides with Satellite laser Ranging, Journal of geodesy, 1997.

Zumberge, J.F., Heflin, M.B., Jefferson, D.C., Watkins, M.M., Webb, F.H., Jet Propulsion Laboratory IGS Analysis Center 1994 Annual report, in IGS 1994 Annual report, IGS Central Bureau, JPL, Pasadena, CA, 1995, JPL Publication 95-18.